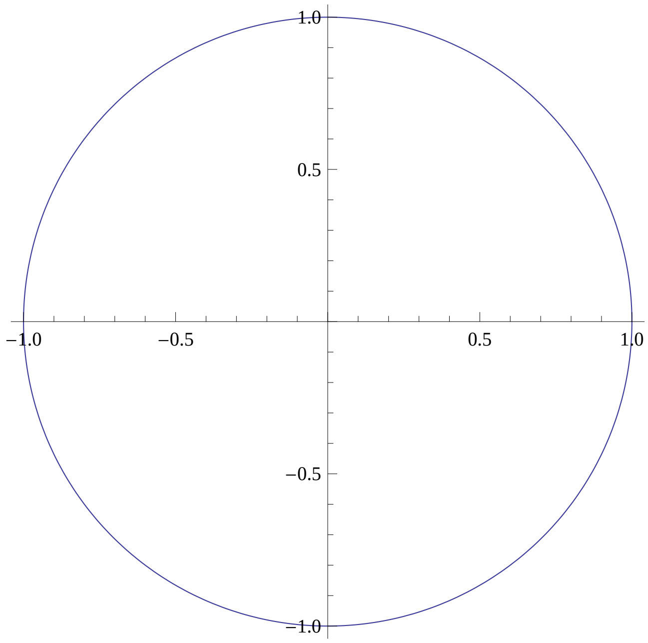
We begin by defining the area of a circle, in polar coordinates.

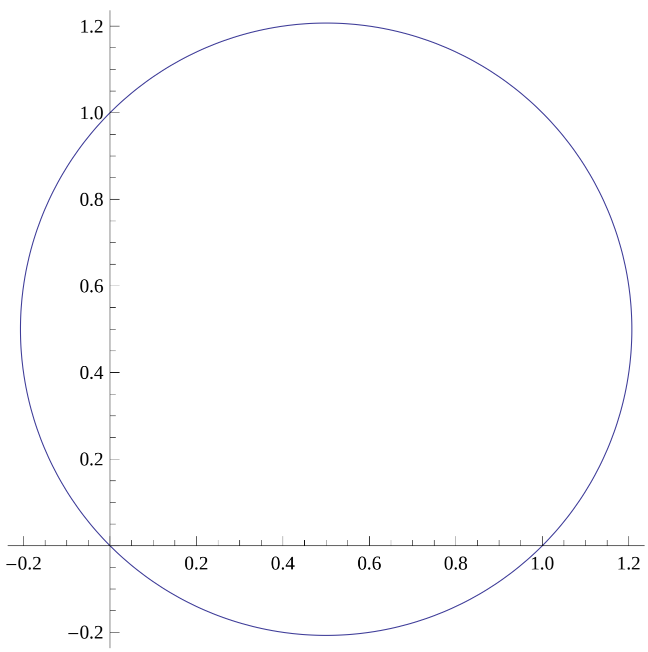
In[259]:=
```
r[θ_] = 1;
PolarPlot[r[θ], {θ, 0, 2 π}]
```



Here I define a second circle, with different boundaries. The equation is not important.

```
r2[θ_] = Cos[θ] + Sin[θ];
PolarPlot[r2[θ], {θ, 0, π}]
```

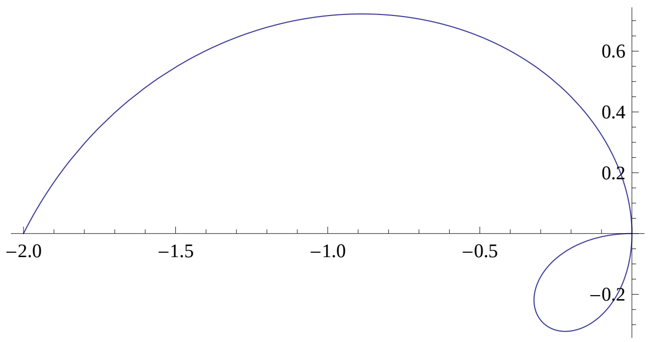
Out[258]=



If we compare the two circles by subtracting the equations, we can see where their radii are equal: at 0 on this graph.

In[212]:= `PolarPlot[r[θ] - r2[θ], {θ, 0, π}]`

Out[212]=



There are two places where the radius is 0, when θ (the angle) is equal to 0, and $\frac{\pi}{2}$.

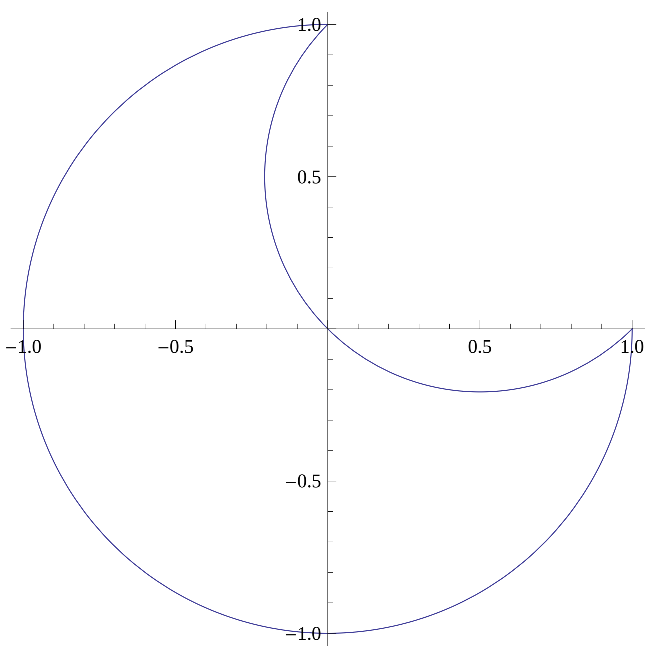
In[228]:= `Solve[r[θ] - r2[θ] == 0, θ]`

Out[228]= $\left\{\{\theta \to 0\}, \left\{\theta \to \frac{\pi}{2}\right\}\right\}$

The new defined shape is shown below, where the second function creates an indent in the first circle. Because the area will always be an angle of 2π, we can use this method to define the shape. We simply subtract or add as needed to the current shape. Where the angles are equal (i.e., where the shapes intersect), we define intervals, and on those intervals use stored shapes to define the addition or subtraction of that shape. The below's shape intervals are $\{\mathrm{r}(\theta) \mid \frac{\pi}{2} \le \theta \le 2\pi\}$, $\{\mathrm{r2}(\theta) \mid 0 \le \theta \le \frac{\pi}{2}\}$.

In[233]:= `Show[PolarPlot[r[θ], {θ, π / 2, 2 π}], PolarPlot[r2[θ], {θ, π / 2, π}]]`

Out[233]=



In[237]:= $\int_0^{2\pi} \frac{1}{2} r[\theta]^2 \, d\theta - \int_0^{\pi} \frac{1}{2} r2[\theta]^2 \, d\theta$

Out[237]= $\frac{\pi}{2}$

Here we calcuate the area of the shape.

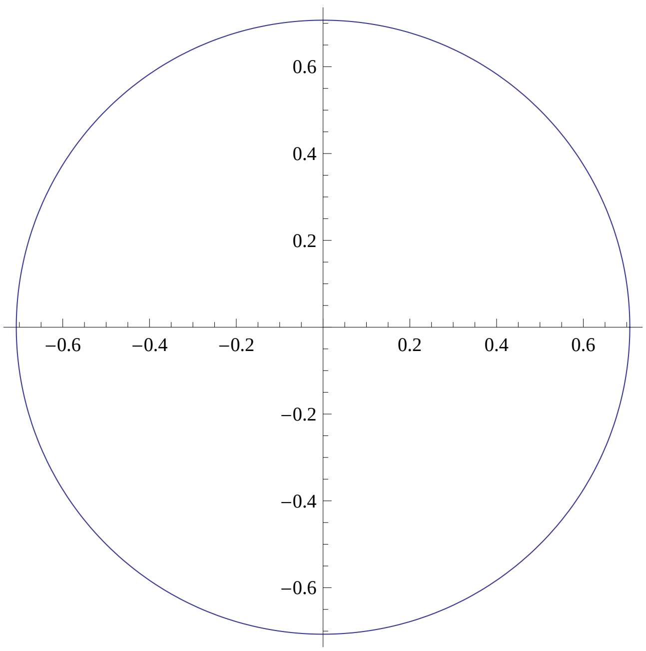
In[266]:= $\text{Solve}\left[\int_0^{2\pi} \frac{1}{2} r^2 \, d\theta == \frac{\pi}{2}, r\right]$

Out[266]= $\left\{\left\{r \to -\frac{1}{\sqrt{2}}\right\}, \left\{r \to \frac{1}{\sqrt{2}}\right\}\right\}$

In[254]:= $r4[\theta\_] = \frac{1}{\sqrt{2}};$

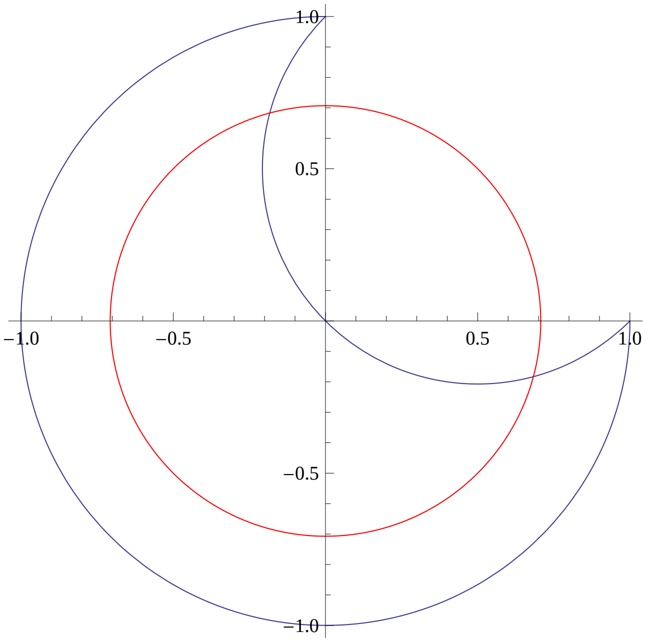
```
PolarPlot[r4[θ], {θ, 0, 2 π}]
```

Out[255]=



In[261]:=
```
Show[PolarPlot[r[θ], {θ, π / 2, 2 π}], PolarPlot[r2[θ], {θ, π / 2, π}],
  PolarPlot[r4[θ], {θ, 0, 2 π}, PlotStyle → Red]]
```

Out[261]=



Finally I've calcuated a circle whose area is equal to the area of the irregular shape— just for kicks.