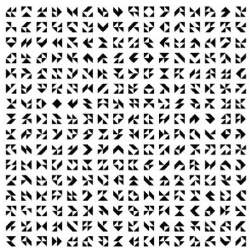
GE 210 Probability & Statistics

Lecture 8: Probability III

September 23rd 2009



All possible permutations of 4 triangles placed in random order.

Today

- Permutations (revisited)
- Combinations
  - Permutations and combinations examples

Permutations (revisited)

Permutations into classes

- Last day, we were looking at selecting items from the sample space that were all distinct. In some cases, this isn't true
  - Ex: List the permutations of the letters a, x, and x

    axx, axx, xax, xxa, xax, xxa

    same   same   same

  - There are only three distinct permutations instead of 6

Permutations (revisited)

Permutations into classes

Theorem: The number of distinct permutations of n objects of which n_1 are of one kind, n_2 of a second kind, and n_k of the k^{th} kind is:

$$\frac{n!}{n_1!n_2!n_k!}$$

Check for the last case (permutations of a, x, x)

n = 3

n_1 = 1

n_2 = 2

$$\frac{3!}{1!2!} = \frac{(3)(2)(1)}{(1)(2)(1)} = 3$$

Example 1: Permutations into classes

- A machinist produces 22 items during a shift. Three of the 22 items are defective and the rest are not defective. In how many different orders can the 22 items be arranged if all the defective items are indistinguishable and the non-defective items are indistinguishable?
- What if the defective items are distinguishable?

Combinations

- When were are interested in selecting r objects from n but order is NOT important, these selections are called combinations
  - Ex: Select 2 letters from a, b, c

    ab, ac, bc, ba, ca, cb

Combinations

- Combinations of n objects taken r at a time usually written as ${}_nC_r$

Theorem: The number of combinations of n distinct objects taken r at a time is

$$ {}_nC_r = \frac{n!}{r!(n-r)!} = \frac{{}_nP_r}{r!} $$

Example 2: Combinations

- A young boy asks his mom to bring 5 video games for their upcoming trip from his collection of 10 arcade and 5 sports games. How many ways are there for his mother to bring 3 arcade and 2 sports games?

Example 3: General Example

- A certain dress shirt comes in five different styles with each style available in four distinct colours. If the store wishes to display shirts showing all the various styles and colours, how many different shirts would it have to display?

Example 4: General Example

- How many ways can five positions on a basketball team be filled with team members who can play any of the positions?

Example 5: General Example

- From a group of 4 men and 5 women, how many committees of size 3 are possible
  - A) with no restrictions
  - B) with 1 man and 2 women
  - C) with 2 men and 1 woman if a certain man must be on the committee?

Example 6: General Example

- A bench can seat 4 people. How many seating arrangements can be made from a group of 10 people?

Example 7: General Example

- A student is to answer 7 out of 9 questions on a midterm test. How many examination selections are there if
  - A) there are no restrictions?
  - B) the first 3 questions are compulsory?
  - C) if the students must answer at least 4 of the first 5 questions?

Example 8: General Example

- How many different 5 card arrangements are there from a standard 52 card deck?
- How many different arrangements of 5 cards are there if you have 2 aces and 3 jacks?

Permutations

Summary

- Permutations depend on selection without replacement
  - Ex: letters could only be chosen once
- Order is important
  - A,B is NOT equivalent to B,A
- If all items are distinguishable

$$_nP_r = \frac{n!}{(n-r)!}$$

- If the items are not distinguishable and you are using all items

$$\#\text{ of permutations} = \frac{n!}{n_1!n_2!n_k!}$$

Combinations

Summary

- Combinations also depend on selection without replacement
- Order is NOT important
  - A,B is equivalent to B,A
- Therefore, fewer possible outcomes than equivalent permutations

$$ {}_{n}C_{r} = \frac{n!}{r!(n-r)!} = \frac{{}_{n}P_{r}}{r!} $$

Next day

- Probability of an event
- More examples