GE 210 Probability & Statistics

Lecture 7: Probability II

September 21st 2009

Today

- Multiplication rule
- Permutations and combinations

Probability

Counting

- The probability of an event occurring depends on the number of outcomes of the event
- If the number of outcomes of an event is large, we will need methods to count the number of these potential outcomes without actually listing them in a tree diagram
  - Ex: An experiment to examine canola yield has 3 different seeds, 5 types of soil, 2 moisture contents, 3 seeding methods and 3 replications
- Method one: Multiplication Rule

Multiplication Rule

- If an operation can be performed in n_1 ways and for each of these a second operation can be performed in n_2 ways, then the two operations can be performed together in $n_1 n_2$ ways.

Example 1: Multiplication Rule

- A developer of a subdivision offers new homeowners a choice of house style: bungalow, split, duplex, and two-storey. They also offer a choice of exterior siding of stucco or siding. How many different options does a new buyer have for a house style and exterior?

Generalized Multiplication Rule

- If an operation can be performed in n_1 ways and if for each of these a second operation can be performed in n_2 ways, and for each of the first two, a third operation can be performed in n_3 ways, and so forth, then the sequence of k operations can be performed in $n_1 n_2 n_3 \ldots n_k$ ways.

Example 2: How many numbers?

Selection Without Replacement

- How many 4 digit numbers can be formed from the numbers 1, 2, 3, 4 if the numbers can only be used once?
- How many 8 digit numbers can be formed from the numbers 1, 2, 3, 4, 5, 6, 7, 8 if the numbers can only be used once?
- How many 3 digit numbers can be formed from the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 if the numbers can only be used once?

Example 3: Four Digit Arrangement

- How many even four-digit numbers can be formed from the digits 0, 1, 2, 5, 6, 9 if each digit can be used only once?
- How many numbers less than 600 can be formed?

Permutations

- A permutation is the arrangement of all or part of a set of objects where the order of the objects is important
- Consider the letters a, b, and c.
  - Possible permutations or arrangements of the letters:

    abc, acb, bac, bca, cab, cba

  - There are 6 distinct arrangements

Permutations

- For the letters a, b, and c we had
  - $n_1 = 3$ choices for the first letter
  - $n_2 = 2$ choices for the second letter
  - $n_3 = 1$ choice for the third letter
  - $n_1 n_2 n_3 = (3)(2)(1) = 6$ permutations
- For n objects, they can be arranged in

  (n)(n-1)(n-2)…(2)(1) = n! ways

Theorem: The number of permutations of n distinct objects taking one at a time (and using them all) is n!

Permutations

- How many arrangements of two letters from the larger group of the letters a, b, c, and d are there? How many permutations are there?

  ab, ac, ad, ba, bc, bd, ca, cb, cd, da, db, dc

- By inspection, there are 12 permutations

Permutations

- When picking 2 letters from 4 choices, we had
  - $n_1 = 4$ choices for the first letter
  - $n_2 = 3$ choices for the second letter
  - $n_1 n_2 = (4)(3) = 12$ permutations
- This can be expressed mathematically as

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

  - Where n = total number of items

    r = how many items are taken out at a time

Permutations

- When picking 2 letters from 4 choices
  - n = 4, r = 2

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

$$ {}_4P_2 = \frac{4!}{(4-2)!} = \frac{(4)(3)(2)(1)}{(2)(1)} = 12 $$

Theorem: The number of permutations of n distinct objects taken r at a time is $ {}_nP_r = \frac{n!}{(n-r)!} $

Example 4: Permutations 1

- Three awards (research, teaching, and service) are given each year to the graduate students in the M.Sc. in Civil Engineering program. There are currently 25 M.Sc. Students enrolled in the program. If each student can receive at most one award, how many possible selections are there?

Example 5: Permutations 2

- A president and a treasurer are to be chosen from a student club consisting of 50 people. How many different choices of officers are possible if:
  - 1) there are no restrictions
  - 2) A will serve only if he is president
  - 3) B and C will serve together or not at all
  - 4) D and E will not serve together

Next day

- Combinations
  - Permutations and combinations examples