The Raffle

Mini Moose 2707
Staredit Network
http://staredit.net

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1 The First Raffle

When SEN developed its first raffle, it was called the "50/50 Raffle". It had a starting jackpot of 50 minerals and added 10 minerals to the jackpot per entrant, which was half of the 20 minerals needed to enter, hence its name. Let n be the number of participants in the raffle. We note $n \geq 1$ because there must be at least one participant in order for the drawing to occur, so division by n is defined. When the raffle winner is drawn, the following can happen to each participant:

(i) Winning: $50 + 10n - 20 = 30 + 10n$ minerals with probability $\frac{1}{n}$.

(ii) Losing: -20 minerals with probability $\frac{n-1}{n}$.

We note that the sum of the probabilities $\frac{1}{n} + \frac{n-1}{n} = 1$, which is necessary for the probability to be valid. (The actual definition is more technical and beyond the scope of this article.) We calculate the expected value or average returns of playing the raffle by denoting the change in a particpant's minerals by X and working as follows:

$$E[X] = \frac{1}{n}(30 + 10n) + \frac{n-1}{n}(-20)$$

Multiplying in the second fraction and adding over the common denominator:

$$E[X] = \frac{30 + 10n - 20n + 20}{n}$$

Combining like terms:

$$E[X] = \frac{50 - 10n}{n}$$

Distributing the division:

$$E[X] = \frac{50}{n} - 10$$

So we note that the expected value or average returns from playing the raffle decreases as n increases, because $50/n$ will decrease. The average returns are zero when there are five participants in the raffle, and then the average returns are negative when $n > 5$. (Which is something we want for a good casino!) In this raffle, it is theoretically to a person's advantage to enter when there are 4 or fewer participants. (and theoretically not detrimental if they are the fifth entrant.)

Now we will attempt to generalize the raffle:

2 Generalized Raffle

Now we will consider a generalized raffle for all variable amounts of starting jackpots, ticket costs, and jackpot increase per ticket. We introduce them as variables:

Let n denote the number of participants in the raffle, as before. $n \geq 1$.

Let p denote the amount paid to enter the raffle. We assume $p > 0$, otherwise the raffle pays participants to enter.

Let a denote the amount added to the jackpot per ticket. We assume $a \geq 0$, otherwise tickets purchased will subtract from the jackpot.

Let s denote the starting jackpot.

We spell out the possible outcomes as before using our new variables:

(i) Winning: $an + s - p$ minerals with probability $\frac{1}{n}$.

(ii) Losing: $-p$ minerals with probability $\frac{n-1}{n}$.

We calculate expected value of the change in a partipcant's minerals as before by multiplying each outcome by its probability and summing them together.

$$E[X] = \frac{1}{n}(an + s - p) + \frac{n-1}{n}(-p)$$

Multiplying the second fraction and adding over the common denominator:

$$E[X] = \frac{an + s - p - np + p}{n}$$

Combining like terms and rearranging:

$$E[X] = \frac{s + an - np}{n}$$

Distributing the division:

$$E[X] = \frac{s}{n} + (a - P)$$

We note that with $s = 50, a = 10, p = 20$, we get the expected value of the original 50/50 Raffle calculated above. We make the following observations:

1. The expected value of playing the raffle will always depend on the number of participants unless there is no starting jackpot. Curiously, a negative starting jackpot creates a raffle where the average returns of the players *improve* as more people participate. (Because s/n would be negative and decreasing n would increase it.)

2. We note the break-even point for average returns is $n = \frac{s}{p-a}$, which agrees with our statements about the 50/50 raffle above.

3. If there is no starting jackpot and the amount added to the jackpot per ticket equals the ticket cost ($p = a$), the expected value of the raffle is zero.

To examine another SEN raffle, we examine the case $s = 0, p = 15, a = 10$, which was tested by DevliN. By our formula for $E[X]$ derived above, $E[X] = 10 - 15 = -5$, which means it is theoretically never to anyone's advantage, on average, to enter this raffle.

The current iteration of SEN's raffle uses $s = 0, p = 10, a = 10$. Applying the formula to this case, we get $E[X] = 10 - 10 = 0$, which agrees with our observations above. On average, a participant in this raffle will theoretically break even.